Dynamics of the Free Rise of a Light Solid Sphere in Liquid

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It has long been believed that the free rise of a particle with a density smaller than that of the surrounding fluid should obey the laws of free settling. This kind of particle will be hereafter referred to as a *light particle*, while the opposite case will be termed a *heavy particle*. In principle, the nature of the forces acting on both free rising and free settling spheres is the same and the only difference is in the direction of the net force (upwards in the first case and downwards in the second). Therefore, one should expect that two spheres (light and heavy) with the same diameter (d_p) and equal, but opposite differences between the densities of the particle and the fluid ($\Delta \rho$), surrounded by the same fluid, should have the same terminal velocities (in opposite directions).

However, it has been shown recently that both the terminal velocity and the trajectory of free rising and free falling particles are different (Karamanev and Nikolov, 1992). The drag curve of a free falling heavy sphere and that of a free rising light sphere are shown in Figure 1. Similarity between the motion of light and heavy spheres was observed when the terminal Reynolds number was smaller than 130 *and/or* the particle density was larger than 0.9 g/cm^3. In these cases the terminal velocity of a light sphere could be described by the standard drag curve. The particle indeed rose following a rectilinear trajectory. When *Re* was greater than 130 *and* the particle density was below 0.3 g/cm^3, a significant difference (up to 2.3 times) between the drag coefficients of free rising and falling spheres was observed. Light spheres rose following a spiral trajectory. Unfortunately, only particles with a density below 0.3 and above 0.9 g/cm^3 were studied.

The difference between both drag curves can be possibly explained by the effect of turbulence on the particle. Light particles have smaller mechanical inertia, and therefore their motion should be affected strongly by the wake behind the moving particle. The rotation of the wake (Achenbach, 1974; Clift et al., 1978) eventually induces the sphere to follow a spiral trajectory. This explanation, however, has not been proven experimentally.

Since the drag coefficient of light particles with densities much smaller than that of the surrounding liquid does not obey the standard drag curve while the drag curve of rising particles with densities close to that of the liquid is similar to the standard drag curve, it is important to study the dynamics of the rise of light particles with a density between 0.3 and 0.9 g/cm^3 and to find the conditions of the transition between these two types of particle movement.

The main objective of this work is to study the effect of the diameter and density of a rising sphere on the drag coefficient and to determine the conditions at which the above mentioned transition occurs. The effect of the wake on the trajectory of rise will be also studied. The results are important for better understanding of processes in which free solid spheres are surrounded by fluid with higher density, for example, in ore treatment (flotation), biotechnology (inverse fluidized-bed bioreactors), geophysics (rise of meteorological balloons), and so on.

Materials and Methods

The experiments were performed in a vertical glass tube with a flat bottom filled with distilled water with a temperature of 25°C. The inner diameter of the tube was 0.3 m, and the height was 1.20 m. The minimal ratio between the col-

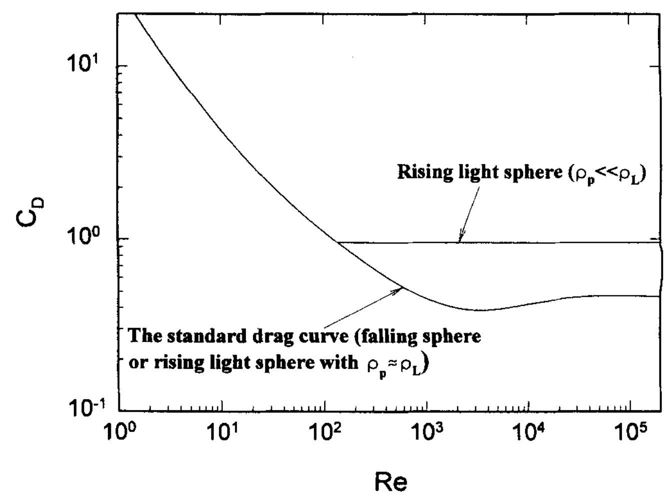

Figure 1. Standard drag curve and drag curve of free rising light particles.

Adapted from Karamanev and Nikolov (1992).

umn and particle diameter was 8:1; therefore, the wall effect was insignificant (Filderis and Whitmore, 1961). The light particles were delivered to the bottom of the column by means of a special rubber clamp and released after a 5-min period necessary for water to become quiescent. The terminal velocities and trajectories of the rising particle were measured using a photographic technique described earlier (Karamanev and Nikolov, 1992). Only the velocities below 10 cm/s were measured by a stopwatch.

Expanded polystyrene (styrofoam) spheres with diameters between 3.3 and 30 mm were used. The density of styrofoam is around 50 kg/m^3. The density of a sphere with a given diameter was varied by inserting stainless steel rods or wires with lengths equal to the sphere diameter and different diameters into the spheres. Special attention was paid to avoid displacement of the center of mass from the geometrical center of the sphere. The inhomogeneity of particle density and momentum about its diameter was found to be insignificant since no rotation around its center and no preferred orientation during the particle rise were observed. The aspect ratio of the particles was always higher than 0.92. It has been shown earlier (Karamanev, 1994) that both the terminal velocity and trajectory of spheroids with this aspect ratio are the same as these of true spheres. The hydrodynamic smoothness of the particles was achieved by the following procedure: (1) a hole with a diameter equal to the rod diameter was drilled through the center of the particle; (2) the rod was inserted, its edges shaped as spherical segments; (3) the edges of the rod were covered by a 100-μm-thick elastic rubber sleeve when the sphere diameter was larger than 7 mm. Using this technique, spheres with densities ranging from 50 to 990 kg/m^3 were produced. The density of spheres with diameter over 10 mm was measured directly from their mass and volume, calculated from the diameter. The latter was measured by a micrometer. The density of smaller spheres was determined using a pycnometer. When the particle density was above 850 kg/m^3, it was measured by a technique described by Karamanev and Nikolov (1992).

Results and Discussion

Drag coefficient and the terminal velocity

The terminal velocity of a light sphere as a function of the sphere density at a constant diameter was studied (Figure 2). It can be seen that in general the terminal velocity decreases with an increase of the sphere density. There is, however, a small part of the curve (corresponding to particle densities between 0.42 and 0.63 g/cm^3) where the terminal velocity remains constant with increase of the particle density. This can be explained by a change in the mechanism of particle-fluid interaction. Indeed, it was observed that the sphere moved via a rectilinear path when its density was below 0.42 g/cm^3, while it rose via a spiral trajectory when the density was above 0.63 g/cm^3. This change can be explained by the interaction between the rising particle and the wake behind it. When the particle density is high, its mechanical inertia is large and the wake rotation does not have a significant effect on the particle motion. With a decrease of the density below a certain critical level, the mechanical inertia of the particle becomes small enough for the wake to induce rotation of the particle, thus creating a spiral trajectory. This results in a significant

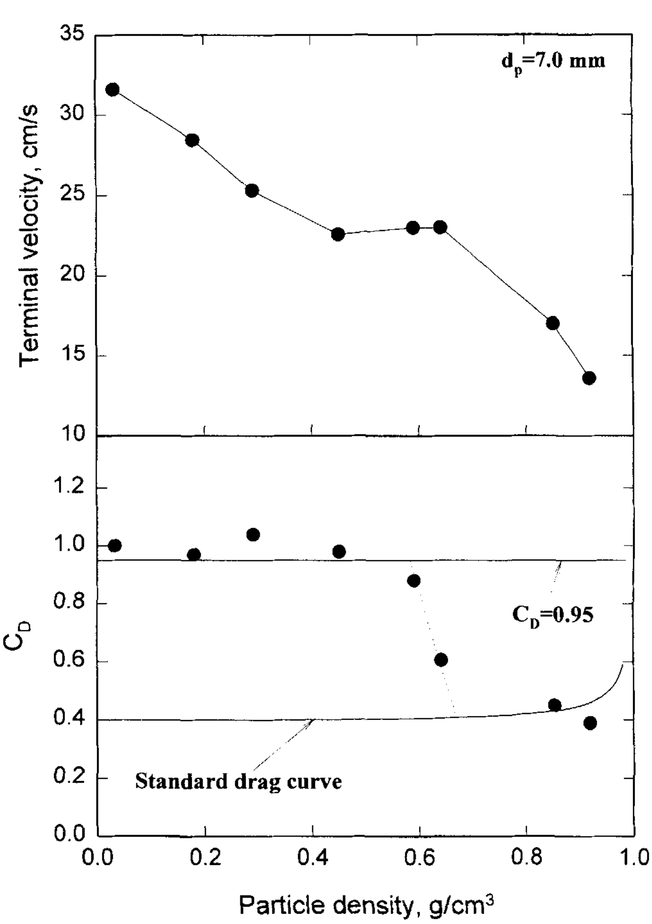

Figure 2. Effect of the particle density on its terminal velocity and drag coefficient.

increase in the drag coefficient (Figure 2). The drag coefficient is close to 0.95 when the particle density is below 0.6 g/cm^3. The theoretical dependence between the drag coefficient and the particle density, corresponding to the standard drag curve, was calculated using the correlation of Turton and Levenspiel (1986)

$$C_D = \frac{24}{Re}(1 + 0.173\,Re^{0.657}) + \frac{0.413}{1 + 16{,}300\,Re^{-1.09}} \qquad (1)$$

Similar curves were obtained with light spheres with diameters between 3.3 mm and 30 mm. The particle density, at which the drag coefficient "jumps" from $C_D = 0.95$ to the standard drag curve, is different for different sphere diameters. The Reynolds and Archimedes numbers, at which this transition occurs, are called the limiting Reynolds ($Re_{\lim}$) and Archimedes ($Ar_{\lim}$) numbers. The relationship between the drag coefficient and Reynolds number for different sphere diameters is shown in Figure 3. The Reynolds number for each sphere diameter was varied by varying the sphere density. It can be seen that $Re_{\lim}$ increases with increasing particle diameters. The transition between the standard drag curve and $C_D = 0.95$ is always sharp, which is within very narrow range of Re.

The influence of different physical parameters of particle and liquid on $Re_{\lim}$ and $Ar_{\lim}$ was studied. It has been found

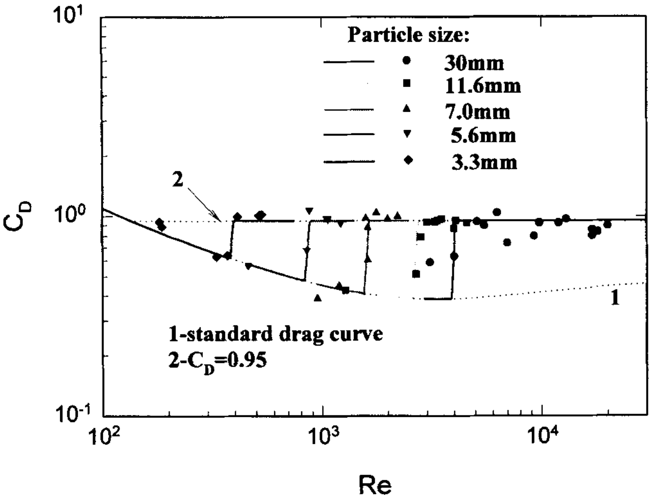

Figure 3. Dependence between C_D and Re for particles with different diameters and densities.

that the best correlated parameters are the particle diameter vs. Re_{lim} and the particle diameter vs. Ar_{lim}. The effect of the particle diameter on Re_{lim} is shown in Figure 4. The least-square fit regression shows that the relationship be-

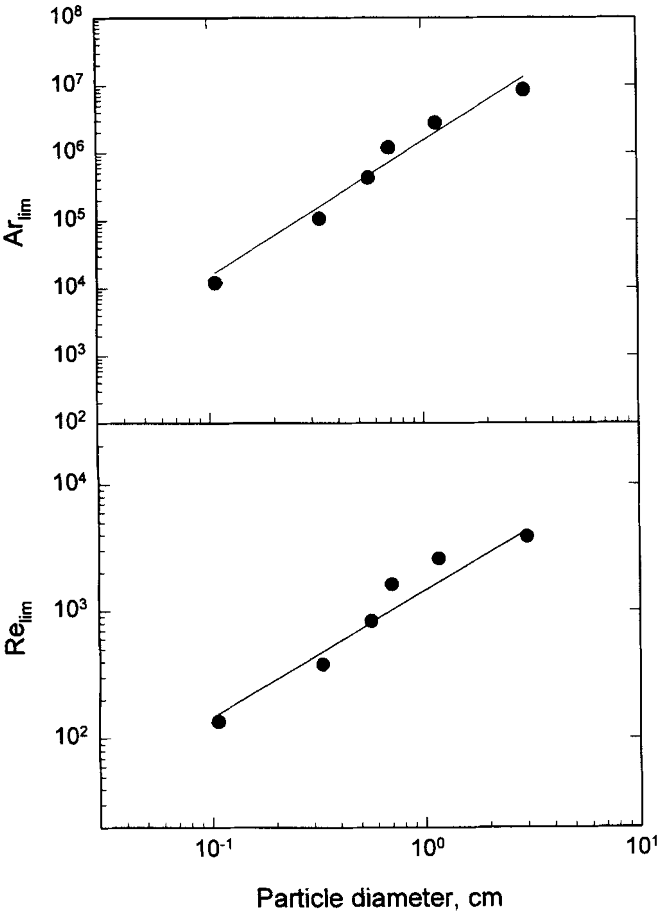

Figure 4. Effect of particle diameter on the limiting Reynolds and Archimedes numbers.

tween these two parameters can be described by the following equation

$$Re_{\text{lim}} = 1{,}450\, d_p \tag{2}$$

A very interesting conclusion can be drawn from Eq. 2. The transition between both flow regimes always occurs at a constant terminal velocity (independent of the particle diameter or density) equal to 14.5 cm/s when the particle rises in water at 20°C. From the point of view of the particle parameters, the transition occurs at $(d_p \Delta\rho) = 0.120$ g/cm^2.

The effect of the particle diameter on the limiting Archimedes number is also given in Figure 4. The experimental data can be described by the following correlation

$$Ar_{\text{lim}} = 1.18 \cdot 10^6\, d_p^2 \tag{3}$$

d_p in Eqs. 2 and 3 is in cm.

Trajectory of the particle motion

The trajectory of the rising particle was also studied. It has been found that for all the particles studied, the light sphere rises via a vertical straight line when the drag coefficient corresponds to the standard drag curve. However, when C_D is equal or close to 0.95, the sphere rises via a spiral trajectory. Each of the spirals had quite a regular shape with a nearly constant wavelength and diameter. The spiral motion of the light spheres will be studied next.

Periodic processes can be characterized by the dimensionless Strouhal number defined as

$$Sr = \frac{fd}{U} \tag{4}$$

It is important to determine which physical parameters are to be used in Sr. When the vortex shedding due to the fluid flow around a fixed sphere is studied, f is the frequency of shedding and U is the relative particle-fluid velocity. The characteristic length d is the sphere diameter. For the case of spiral rise of a solid sphere, f should be the frequency of the particle rotation around the spiral axis and U should be the terminal velocity U_t. The main question is which parameter is to be used in Sr as a characteristic length. In the case of a fixed sphere, particle-wake interactions take place only in the vicinity of the sphere surface and the sphere diameter is the main space parameter. However, the sphere rising by a spiral path makes the wake to rotate around the spiral axis together with the sphere. Therefore, it seems logical to propose that the spiral diameter d_{sp} is the main space parameter of the rising light spheres and should be included in Strouhal number as a characteristic length. For the case of a spiral motion

$$f = \frac{U_t}{h_{sp}} \tag{5}$$

and therefore, Strouhal number becomes equal to

$$Sr = \frac{d_{sp}}{h_{sp}} \tag{6}$$

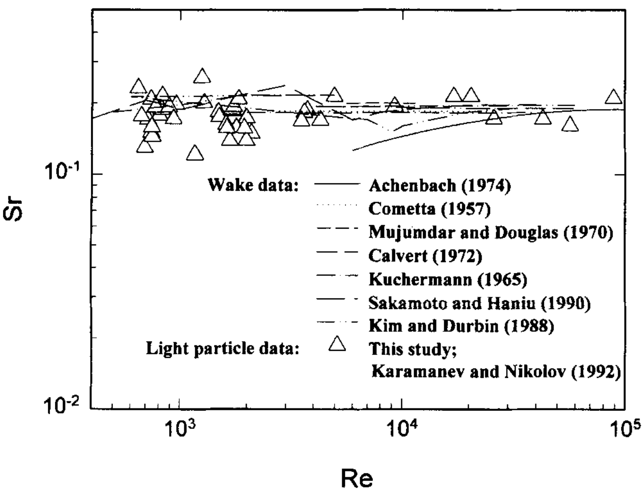

Figure 5. Strouhal number vs. Reynolds number for the wake behind a fixed sphere and free rising light particle.

where h_{sp} is the spiral wavelength. The effect of the Reynolds number on Sr is shown in Figure 5. The experimental data used in the figure are both from this study as well as from a previous work (Karamanev and Nikolov, 1992). It can be seen that Sr of the spiral movement of the rising light sphere is a constant, independent of the Reynolds number. The experimental lines of the effect on Re on Strouhal number of the vortex shedding behind a fixed sphere (Achenbach, 1974; Cometta, 1957; Kuchemann, 1965; Kim and Durbin, 1988; Sakamoto and Haniu, 1990; Calvert, 1972; Mujumdar and Douglas, 1970) are also shown in Figure 5. It can be seen that the wake Strouhal numbers are exactly the same as those for the spirals of the rising particles. This shows the close link between the vortex shedding and the spiral motion of free rising particles.

Conclusions

The results of the present work showed the existence of two different regimes of the free rise of light solid spheres. The first regime is characterized by a rectilinear motion of the solid sphere. The drag coefficient follows the standard drag curve. This regime is observed when $Re/d_p < 1{,}450$ (corresponding to $Ar/d_p^2 < 1.18 \times 10^6$). When the fluid is water, the transition always occurs at a constant terminal velocity $U_t = 14.5$ cm/s. The second regime is observed when $Re/d_p > 1{,}450$. In this case the sphere follows a spiral path. The drag coefficient is constant and equal to 0.95.

It has been shown that the Strouhal number of the spiraling sphere is equal to that of the vortex shedding behind a fixed sphere. This finding shows that the wake rotation is the main factor for the spiral motion of the rising sphere.

Notation

Ar = Archimedes number defined as: $Ar = d_p^3 g |\rho_f - \rho_p| \rho_f/\mu^2$
$Ar_{\lim}$ = limiting value of Ar when particle motion transition occurs
f = frequency of a periodic process
g = gravity acceleration
Re = particle Reynolds number defined as: $Re = U d_p \rho_f/\mu$
$Re_{\lim}$ = limiting value of Re when particle motion transition occurs
U = particle-fluid velocity

Greek letters

μ = fluid viscosity
ρ_f = fluid density
ρ_p = particle density

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